Burns–Wilkinson: scoped audit and repair checks

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Completed scoped audit. Six independent source-only assessments and two focused Astra 6.2 challenges have completed, all without truncation. The record distinguishes checked defects, proposed repairs, conditional imported results, and unchecked parts. This is an AI-assisted source audit, not a human certification or a complete verification of every cited theorem. Publication authorized on 21 November 2026.

The target is Burns and Wilkinson, *On the ergodicity of partially hyperbolic systems*, Annals of Mathematics 171 (2010), 451–489, publisher record and paper, together with the separate June 2005 preliminary note *A note on stable holonomy between centers*. The journal references the note under a slightly different title and a 2006 date. Those versions are not silently identified.

1. Present conclusion

There is a demonstrably false auxiliary estimate in the supporting note, and there are local formulation and proof defects in the published Annals paper. Selected later papers do reuse affected inputs or arguments. This campaign has **not found a counterexample to the principal ergodicity theorems**. The available evidence supports a substantially narrower claim than saying that the main theorems and their descendants are false.

The two regularity branches matter. The C2 branch uses a corrected Pugh–Shub–Wilkinson input. The C1+delta branch uses the separate BW note and stronger bunching assumptions. A defect in the latter input does not remove the former route. Local density arguments are shared and need their own repairs.

It would also be wrong to say that the holonomy problem went unnoticed. Wilkinson's publication list acknowledges errors in the note and directs readers to Brown. Brown and Saghin provide later repair routes with different stated hypotheses and scope; matching those routes to the BW construction was a specific audit obligation. The focused check now supports the Saghin substitution at the original bunching strength, conditional on that imported result and the graph construction.

2. Confirmed local defects

Supporting-note Lemma 1.1(2)

The iterated estimate fails even for a smooth linear product with constant center distribution. Let

$$A = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}, \quad f = A \times \mathrm{id}_{S^1}, \quad \lambda = (3 - \sqrt{5})/2.$$

Use the flat product metric, take $\mu = \hat{\mu} = 3/10$, $\nu = \hat{\nu} = 2/5$, $\gamma = \hat{\gamma} = 9/10$, $\theta = 1/2$, and regularity exponent $\delta = 3/4$. These satisfy the strict rate and bunching inequalities. For any proposed lemma radius choose a smaller fixed $\eta > 0$. The two base points 0 and $\eta\lambda^n e_s$, with identical center directions, remain within that radius for all backward times from zero through n. The asserted estimate would give

$$\eta \le \sqrt{\eta} \left(\frac{50(\sqrt{5} - 1)}{81} \right)^n \longrightarrow 0,$$

an impossibility. The coordinator checked the argument and exact rational inequalities; both source-only providers independently found counterexamples to this estimate. Astra additionally checks that both time directions, including the lemma's extra small-initial-distance clauses, fail. The exact-check bundle contains a portable Python script and its recorded output. It checks the scalar inequalities; the general geometric argument is the one given here.

This example is not essentially accessible and its actual center holonomies are smooth. It refutes the auxiliary estimate, **not** the note's regularity conclusion or the ergodicity theorem. The note's later convergence steps cannot simply cite the estimate as printed. The two initial reviewers propose different replacement arguments. Astra has now checked Sol's actual-linear-map argument for the compact coherent note theorem; the coordinator accepts the local argument with the scope and premises described in Section 5 below. Fable's different proposed repair has not had a separate focused adjudication.

Journal Proposition 2.4, printed p. 462

The definition of a regular family on p. 459 includes nesting and positive ambient volume. The fiber-ratio hypotheses in Proposition 2.4 do not imply either condition. Alternating equal disjoint rectangles in a vertical foliation violates nesting while satisfying the ratios. A shrinking vertical interval over a singleton transversal base satisfies the inequalities but has zero ambient volume.

Adding nesting and positive-volume assumptions makes the printed integration argument prove the intended successive-volume bound. Both initial providers find that the concrete BW applications supply the needed extra conditions. Astra checks the BW applications and the corrected abstract integration proof. For ABW, it identifies a specific one-step nesting condition not present in its supplied excerpts. The coordinator then checked the additional primary pages: Proposition 5.4(3)(b), printed p. 950, gives the moving-chart inverse unstable contraction, while (5.8) and the following display, p. 957, give $\tau\hat{\nu} \prec \sigma < 1$. Hence the inverse image of the terminal radius τ_{n+1} is contained in the preceding radius τ_n, on the controlled local plaques. Lemma 5.8 on p. 957 supplies the shrinking working-neighborhood control. These statements discharge the identified nesting condition on their geometric premises; their entire upstream nonuniform graph construction has not been re-proved. This is a local overstatement, not a failure of the underlying integration estimate.

Journal Lemma 8.2, printed p. 485

The proof uses same-index julienne membership where Proposition 5.3 provides fixed index shifts. It also infers a positive lower stable-fiber length from diameter upper bounds, which do not alone supply that lower bound.

A checked local completion uses stable-coordinate projection in a fixed local chart. The endpoints remain O(sigma_n) close; tangent continuity makes the weak-plaque contribution o(sigma_n), while the stable base-boundary displacement has a positive multiple of sigma_n as its projection. Local product structure identifies fiber boundaries with base boundaries. The boundary lower bound gives an inradius, not merely a lower diameter: an escaping path would cross that boundary. Fixed rank shifts absorb the resulting constants. This is a new justification, not a claim that the missing lower estimate was already proved by the displayed triangle inequalities. Astra and the coordinator accept this local completion for BW on its uniform plaque premises. It also works for ABW in one fixed time-zero adapted chart, without asserting a uniform comparison of all future adapted metrics. The HT transfer remains conditional on its precise history product and base-inclusion inputs.

3. Additional findings from the independent panel

Both providers identify a metric convention needed for the weak cs/cu bounds in Proposition 3.1. Norm estimates on Es, Ec and Eu separately do not imply the same bounds on their sums for a strongly nonorthogonal metric. This can be exhibited by the same product map using nearly parallel stable and center unit vectors in the chosen inner product. An orthogonal product metric on the invariant splitting, followed by a sufficiently close smooth approximation, is a remedy using the strict compact margins. Astra confirms this normalization without changing the bunching hypotheses, smooth measure class or density points. The overstrong metric-dependent clauses must not be used for every metric satisfying only the three separate summand bounds.

The fake-Jacobian application of Lemma 4.4 also needs an explicit orbit-indexed version: a fake distribution depends on its orbit chart and need not define a single function on the original manifold. Uniform Holder bounds on the chart functions give the same summable log-distortion calculation. This is distinct from genuine unstable Jacobians, which do define one function on the manifold.

In Lemma 6.2, genuine stable holonomy cannot be treated as membership in the same fake center-stable leaf. The checked local completion constructs the terminal local-product intersection and pulls it back, checking the working domains and rank shifts. Other recorded discrepancies are orbit subscripts, cocycle base points, and endpoint strict inequalities. A typographical discrepancy is not counted as an independent failure of a main theorem.

Several panel items are **coverage limits**, not established defects. Examples include unprovided graph-transform proofs, full hypotheses of classical absolute-continuity results, and the precise Pinsker input. The initial density packet itself mistakenly requested Lemmas 8.4 and 8.5; Section 8 ends at Lemma 8.3. That was our target-inventory error and has been corrected.

4. Selected downstream dependencies

This is a check of the following mathematical uses, not an exhaustive citation graph or a complete audit of any later paper.

Later source	Source-supported dependence	Scope that must be preserved
Rodriguez Hertz–Rodriguez Hertz–Ures (2008), Proposition B.7, printed p. 374	Cites the BW holonomy note for a lower-regularity strong-subfoliation assertion	Its main Theorem B is C2, and the next paragraph gives a separate Pugh–Shub route for C2 with one-dimensional center.
Avila–Bochi–Wilkinson (2009), Proposition 5.2(1), p. 946	Repeats the fiber-ratio regularity criterion, explicitly citing BW Section 2.3	Check nesting and positive measure in the actual applications; its analytical Theorem B is C2.
Avila–Bochi–Wilkinson, Lemma 5.21, pp. 965–966	Reuses the same-index membership and lower stable-length argument	A fixed-chart repair must work with its adapted metric; C1 in the genericity title is not the differentiability hypothesis of this analytical theorem.
Avila–Santamaria–Viana (2013), Section 2.1, p. 16	Explicitly invokes BW Theorem 0.3 for the blanket strongly bunched C1+alpha extension	Keep that extension separate from its C2 statements.
Hammerlindl–Tyler (2025), Proposition 3.1(vii), pp. 14–15	Imports both regularity clauses through the BW construction	Its main Theorem 1.1 is C2; the additional lower-regularity clause is not the only route to it.
Hammerlindl–Tyler, Lemma 9.4, p. 28, Appendix B, pp. 32–33	Adapts Lemma 8.2 and repeats the unsupported lower-length inference	Unlike BW's paragraph it does invoke a holonomy-control result. The repair must respect projection from the orbit space and local fiber geometry.

No conclusion about the authors' awareness or the falsity of the later main theorems follows from these edges. Other descendants mentioned in the owner's report remain outside the source checks completed here.

5. Historical proof and modern repair

The historical external-input cutoff is the journal's received date, 11 October 2005. The June 2005 preliminary note is before that cutoff. The bibliography's 2006 label does not establish that no earlier note existed. Brown and Saghin are later work and must remain labeled modern replacements.

Brown's retrieved manuscript expressly uses stronger bunching. Its arXiv side stamp and internal date differ; this record preserves the actual frozen file without claiming all of its content existed in 2016. Saghin's 2023 preprint contains uniform C1 fake holonomy as Corollary 1.1 and explains the noncompact uniform setting in Section 3.7. The corresponding journal text labels the result Corollary 1.5. We have the complete preprint PDF and page images; the published PDF could not be downloaded locally and was read only as web text.

Modern replacement of the fake-holonomy input

The focused check supports the substitution at the original BW bunching strength, without Brown's extra numerical assumptions. Its premises are Saghin's uniform fake-holonomy and nearby-pair derivative estimates, and BW's coherent uniform graph construction **before** Proposition 3.1(vii). It does not assume the strong C1 subfoliation that it is meant to supply.

The rate dictionary is $\lambda_s = \nu$, $\lambda_u = \widehat{\nu}^{-1}$, $\lambda_c^- = \gamma$, $\lambda_c^+ = \widehat{\gamma}^{-1}$. The stable and unstable theta-bunching tests are exactly BW's two tests. Its auxiliary rate inequalities force $0 < \theta < 1$; $\theta < \delta$ then gives C1+theta regularity. This avoids an unstated change of exponent.

Fix one cutoff extension of the fiber maps and one associated graph family. The nonlinear error vanishes to first order at zero. Compactness and the strict margins give uniform derivative Holder, inverse derivative, cone and bunching bounds. To check a holonomy away from the zero section, translate the same global fiber dynamics along that point's full orbit. This retains all those bounds and the same graph family; constructing unrelated new fake leaves at each point would not suffice. This is the uniform noncompact setting described in Saghin Section 3.7.

Uniform C1 holonomy in both directions gives upper and lower derivative bounds. For joint continuity, more than a bound on the derivative norm is needed. Saghin's pp. 13–14 give a short-leg derivative estimate relative to projection between the two center spaces; p. 16 supplies the uniform-fiber setting. By equivariance, transporting those projections from a contracting end

of the orbit approximates the holonomy derivative with a uniformly geometric tail. The finite products are continuous; hence so is their uniform limit. The coordinator inspected these primary page images as well as the application argument.

Factor holonomy on a general controlled transversal through center disks at its two crossing points. The crossing maps have derivatives given by linear projection along the continuous strong tangent field. Combining these with the center-to-center derivative gives continuous, uniformly invertible transverse derivatives. In local weak-leaf graph coordinates, the strong coordinate supplies the other derivative of the foliation box. This yields uniform C1 strong-foliation boxes inside the weak leaf and two-sided volume comparison by change of variables and Fubini. These are the inputs needed for center-radius control and fake unstable disintegration in the BW density proof.

This closes the **application check** on the stated imported premises. It does not independently reprove every part of Saghin or the graph-transform theory, prove genuine strong-foliation absolute continuity, or settle the Pinsker argument. It is a modern replacement, not a retrospective validation of the June 2005 proof or a blanket audit of every descendant.

A separate new proof for the compact coherent note theorem

Sol proposed an independent construction, and Astra checked it without using Brown, Saghin or Proposition 3.1(vii). It uses the note's compact coherent geometry and its theta-Holder center bundle. The coordinator checked the linear-map telescoping identity, summability, branch selection and scope. This gives a locally checked replacement argument, not a claim of mathematical priority or human certification.

Buffer the rates so that $\nu < a < \min(1, g)$, $g < \gamma$, $\rho = \sup a/g < 1$, and $b = \sup a^\theta/(g\hat{\gamma}) < 1$. For a short stable leg from x to y=h(x), mark at time n the target center sheet through f^n(y). Intersect it with a short smooth transverse disk through f^n(x). The marked sheet resolves recurrence ambiguity without differentiating h. The local intersection maps are C1. Pulling back their center errors gives $O(\ell a_{0:i}\rho^{n-i})$ at time i; a first-exit argument keeps the arcs in the tubes where the buffered conorm applies.

Write A_i and B_i for center derivatives on the source orbit and the approximating orbit, and J_i for smooth transport followed by center projection. With forward product convention, put

$$R_i = B_i^{-1} J_{i+1} A_i - J_i.$$

If P_n is the terminal intersection map, the chain rule and exact telescoping give

$$B_{0:n}^{-1} DP_n A_{0:n} - J_0 = \sum_{i=0}^{n-1} B_{0:i}^{-1} R_i A_{0:i} + B_{0:n}^{-1}(DP_n - J_n) A_{0:n}.$$

The terms are bounded by $C\ell^\theta b^i$ and the final error by $C\ell^\theta b^n$. A summable bound alone would not establish convergence because the summands depend on n: for every fixed i the approximating orbit also converges uniformly to the true y-orbit. Truncate the series and then use its uniform geometric tail. Thus the actual derivatives converge uniformly, as do the maps, proving C1 holonomy. Short-leg closeness to projection supplies invertibility. This avoids the gap between convergence of projective directions and convergence of actual derivatives.

The finite transported derivative products have Holder constants at most CD^m, while their tails are $O(b^m)$. Balancing these bounds gives a positive Holder exponent, for example $\theta \log(1/b)/(\log D + \log(1/b))$. No original explicit exponent is claimed. Longer fixed local legs are covered by finite compositions; there is no uniform bound asserted for unbounded holonomy length. This compact coherent argument does not itself establish the separate uniform fake-model transfer.

Two classical sources were obtained after the paid requests were frozen and are coordinator-only supplements. Brin–Pesin Proposition 5.1 is indeed on printed p. 203 and is introduced under a smooth invariant measure. Its proof cites Sinai's partition theorems. The extension to all the measure and rate conventions used in BW remains a named external obligation here, not a new counterexample. Pugh–Shub Theorem 2.1, pp. 4–5, gives measurewise C1 strong foliations with a jointly continuous positive Jacobian. In one transversal dimension that conclusion yields C1 holonomy, but applying it on the relevant weak leaves still requires its geometric hypotheses. Full independent proofs of these classical inputs have not been audited.

6. Provenance, cost and remaining work

The owner's consolidated report is preserved but was withheld from the six initial requests. Those requests used primary journal page images, the complete October 2006 text as labeled context, the separate holonomy note, and the PSW correction as appropriate. Sol 6.2 high and Fable 5.2 high each reviewed geometry, density, and the note. This is a two-provider independent panel, not a three-provider panel. Later papers and repair literature entered only the focused challenges. Fable's replies arrived after those challenges were frozen.

All eight assessments are structured and uncapped. Conservative usage totals **USD 9.516245**; the uncached batch-tier estimate is **USD 7.766056**. The initial six cost USD 5.799921 under the conservative accounting, and the two focused checks USD 3.716324. The child allowance was delegated from the original follow-up hold, not additional to it. The campaign ceiling remains **USD 25**. These figures are token-based accounting, not invoices. No inference request remains pending and no further calls are needed to complete this scoped record.

The remaining mathematical limits are explicit: complete imported proofs of graph transforms and strong-foliation measure results; the full Pinsker application; historical note-version collation; HHU's weak-leaf use of its alternate theorem; the full ASV extension; ABW's upstream nonuniform construction; and HT's history-product/base-inclusion and measure arguments. They are not additional demonstrated false statements. Neither a complete proof of either principal theorem nor an exhaustive downstream audit is claimed.

An invalid auxiliary estimate does not refute its consumer theorem.

The neutral reading map indexes 37 numbered statements, 10 sections and 32 bibliography entries. Its 83 nodes include separate regularity branches and imported inputs. The 32 source-attributed edges cover the selected argument, not every mathematical dependence. The panel decision graph, repair records, reviewer responses and cost/coverage record remain separate from that reading map.

7. Sources and downloads

The downloadable main paper is the complete October 2006 author edition (50 pages). The audit's printed page references and primary proof images are from the published Annals edition (2010, pp. 451–489). The extracted source index combines that journal text with the separate June 2005 note. It is a quotation/indexing aid, not native TeX; mathematical page images are authoritative. The differing title/date in the journal's reference to the note is recorded, without asserting that every historical version has been collated.

- Main paper: author edition.
- June 2005 supporting note.
- Source editions and hashes.
- Download this report.
- Exact-check bundle.
- Coverage and costs.
- Repair dispositions.

Later-source checks used the following primary texts: HHU (2008), ABW (2009), ASV (2013), and Hammerlindl–Tyler (2025). The repair comparison used Brown's retrieved manuscript and Saghin's 2023 preprint. The full source record distinguishes the versions actually inspected from published metadata and retains their hashes.

Cite this report

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