Church–Farb: what fails, what survives, and what can be repaired

Igor Rivin · Math Referee · 21 November 2026
Public scoped report. Prepared with independent model assessments, focused challenges and coordinator source checks. These are AI-assisted mathematical adjudications, not an independent human review or a formal proof certificate.

Paper: Thomas Church and Benson Farb, *Representation theory and homological stability*, Advances in Mathematics 245 (2013), 250–314, DOI 10.1016/j.aim.2013.06.016; final author version arXiv:1008.1368v3.

The checked portions contain several false statements, not merely typographical errors. They include nonuniform closure and cancellation assertions, implications about mixed stability, an all-rank stability condition, and the prime-two coefficient assertion in Theorem 8.4. These failures do **not** refute the entire representation-stability framework. Finite-dimensional uniform statements have substantially different hypotheses; important ones admit replacement arguments. The pure-braid argument survived the scoped checks described below.

All eleven submitted requests have returned. Eight contain completed mathematical assessments; three initial Fable attempts exhausted their output allowance. There are no pending model jobs and no further paid calls scheduled. Conservative usage accounting is **$11.963289 of the authorized $25**. The separate uncached Batch-tier estimate is **$10.344260**. Both include failed attempts; neither is an invoice. The source packets, original answers and failures remain preserved.

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The uniform cancellation repair was examined in both focused Astra packets. The eventual and polynomial homology reconstructions were checked by the coordinator against the source and Astra proposals; the full replacements have not received a separate fresh final audit. Their improvement records retain that distinction.

1. Scope and versions

The complete final-author TeX was supplied to the initial panel. Its hash is 5f5f379bd8e23f2d3911be054310baedf23dc35a 7b2693e6d3e96bef44654698. The supplied journal PDF, retained privately, has hash b791b01ac888a0e74341bbf2f2931c eb7335f0ef02fd52e46d1e442573ba08d2. Selected journal passages retain the disputed wording: p.269 (mixed stability and Theorem 3.1), pp.275–276 (Theorem 3.5), pp.286 and 289–291 (Lie coefficients and simple stability), and p.311 (Theorem 8.4). This is a scoped edition comparison, not a complete collation of all 65 journal pages.

The journal records receipt on **19 January 2012**. The initial packets had already frozen a conservative first-arXiv cutoff, 7 August 2010; their prompts were not retroactively changed. The focused packets use the verified received date. In particular Hemmer's 2010 online article and Putman's August 2011 preprint are eligible inputs. Later results are retrospective context, not justification of the original proof. Model training knowledge cannot literally be time-filtered.

Part	Checked outcome	Boundary
Theorem 3.1, nonuniform SL clauses	False	Explicit finite-dimensional examples
Theorem 3.1, finite uniform clauses	Supported on the specified classical operation formulas	External formula proofs not exhaustively audited
Theorem 3.5 and Remark 3.6	Nonuniform conclusions false; projection proof invalid	Finite uniform replacement below
Mixed/dual implications after Definition 2.16	False in the stated generality	The definition itself is not being rejected
Theorem 4.1, pure braids	No defect found in the checked argument	Earlier cohomology inputs not all reproved
Theorem 5.3	Compatible-coefficient version has a replacement argument	Clarify what the transition maps mean
Theorem 5.6 and Corollaries 5.7–5.9	Literal all-rank conclusions fail	Polynomial and eventual versions can survive
Theorem 8.4	False at prime two as stated; exterior-cube table reversed	Corrected full periodicity and map claims remain partly open in this audit
Other claims, especially §7 and other §8 applications	Not finally adjudicated	Initial observations are not accepted findings

2. Nonuniform closure and cancellation

Let $E_n = \mathbb{Q}^n$ and let a_n alternate between 1 and 2. All representations in the following finite-dimensional examples are rational algebraic representations, so restricting to the intended category does not remove them.

For SL_n, E_n^* has label (1^{n-1}), which escapes every fixed label as n grows. Consequently E_n and $(E_n^*)^{\oplus a_n}$ are multiplicity stable, but their tensor product has trivial multiplicity a_n, since $E_n \otimes E_n^* = \mathbf{1} \oplus \mathfrak{sl}(E_n)$. Similarly, $\mathrm{Sym}^2(E_n \oplus (E_n^*)^{\oplus a_n})$ has exactly a_n invariants for $n \geq 3$. Taking one-box inner or outer Schur functors carries these counterexamples to the other nonuniform closure clauses.

A second example exposes unbounded multiplicity rather than oscillation. Set $X_n = \bigwedge^* E_n$. Its two trivial summands and its distinct remaining exterior powers give stable multiplicity at each fixed label, while

$$\dim(X_n \otimes X_n)^{SL_n} = 4 + (n-1) = n + 3.$$

The source of the failure is that moving labels can produce a fixed output: removing a full column changes an SL_n label. Pointwise stabilization does not bound all contributors to a fixed tensor-product constituent.

For Theorem 3.5 take $W_n = \mathrm{Sym}^n E_n$ and $V_n = \mathbf{1}^{\oplus a_n}$. This works for SL, GL and Sp. Both W_n and $V_n \otimes W_n$ are nonzero and every fixed multiplicity eventually vanishes; V_n is not multiplicity stable. Even a fixed nonzero limiting factor is not enough: for SL_n, take $W_n = E_n$ and alternate V_n between $\mathbf{1}$ and $\mathfrak{sl}(E_n)$. The product always contains one standard constituent, and its other labels escape. The unknown factor still fails to stabilize.

Remark 3.6 also fails pointwise. Alternate the trivial module with the traceless adjoint for SL_n or GL_n. The adjoint square has one invariant. Every other fixed ordinary label is eventually excluded by scalar degree for GL_n, or weight-lattice degree modulo n for SL_n. Thus the squares stabilize pointwise while the original trivial multiplicity oscillates. This is a weight-lattice argument, not an assumption about roots of unity in $\mathbb{Q}$.

There is a separate infinite-multiplicity obstruction: a countable direct sum of trivial modules absorbs either one or two copies under tensoring. Ordinary representation-ring cancellation is an integer-coefficient argument and cannot be applied to infinite cardinal multiplicities.

3. A replacement for finite uniform cancellation

The printed rank projection is not a ring homomorphism. For example, its $SL_3 \to SL_2$ projection kills E_3^* and the adjoint but sends $E_3 \otimes E_3^* = \mathbf{1} \oplus \mathfrak{sl}_3$ to $\mathbf{1}$. The compactness argument using high-size labels as an ideal fails for the same reason: a representation and its dual can produce the unit.

Nevertheless, **finite-dimensional uniform cancellation is valid**, assuming the classical stable tensor-product tables for each fixed pair of labels.

Suppose W_n and $C_n = V_n \otimes W_n$ are uniformly multiplicity stable and W_n is eventually nonzero. Their stable supports B, F are finite. Fix a constituent $\mu \in B$. For every constituent λ of V_n, the Cartan constituent $\lambda + \mu$ occurs in C_n. Hence every λ belongs to the fixed finite candidate set obtained by subtracting μ from members of F. For GL_n, keep both the polynomial partition and the determinant exponent; the paper's ordinary labels repeat their last entry with rank.

Beyond the finitely many relevant product-table thresholds, transport the decompositions at two ranks to a single larger rank. They satisfy $xw = yw$ there, with $w \neq 0$. The finite-dimensional representation ring embeds by characters in the integral group ring of a free weight lattice, a Laurent polynomial domain. Thus $x = y$. This proves uniform stability of V_n.

The same argument treats uniformly stable squares: 2λ bounds the possible input support, and $(x-y)(x+y) = 0$ implies $x = y$ for effective classes. The proof uses bounded support before cancellation; it does not validate the printed projection or any nonuniform extension. Fixed-label tensor, restriction and Schur-functor formulas also support finite uniform closure. Their imported classical proofs and sharp ranges were not all re-audited here.

4. Mixed stability and duals

The assertions following Definition 2.16 must distinguish ordinary labels from mixed labels and distinguish multiplicities from transition maps.

Here is an example with actual injective, orbit-generating maps. Put $B_n = \mathbf{1} \oplus E_n^{\oplus a_n} \oplus \operatorname{Sym}^2 E_n$ and $T_n = \det(E_n) \otimes B_n$. Write $E_{n+1} = E_n \oplus \mathbb{Q}z$. At a one-to-two increase in a_n, use

$$(q, v, w) \longmapsto (q, v, qz, w);$$

at a two-to-one decrease, use

$$(q, v_1, v_2, w) \longmapsto (q, v_1, w + v_2 z).$$

Old symmetric tensors and tensors involving z are disjoint, proving injectivity. At an increase, the orbit of $(1, z)$ generates both the trivial and new standard summands by taking differences of translates; the pure old summands supply the others. At a decrease the pure old summands already generate. Twist these maps by the canonical determinant-line inclusion.

Every fixed mixed-label multiplicity of T_n is eventually zero, because its highest weights have a long middle of ones. But the fixed ordinary label $\det(E_n) \otimes E_n$ has multiplicity a_n. Thus mixed stability does not imply ordinary GL stability under the printed naming convention. The analogous implication for SL is not refuted: the empty negative mixed label tests each ordinary normalized SL label.

For the dual assertion use $Z_n = \mathbf{1} \oplus (E_n^*)^{\oplus a_n} \oplus \mathfrak{sl}(E_n)$. The increase map inserts qz^* as the second covector; the decrease map inserts $z \otimes f_2$ into the lower-left block of the new adjoint matrix. Old blocks and the new row are disjoint, and the same orbit argument proves injectivity and generation. Only the trivial ordinary label persists in Z_n, whereas Z_n^* has standard mixed-label multiplicity a_n. Ordinary stability therefore does not imply mixed stability of the duals. Duality does interchange the two mixed labels of an individual simple module; natural transposes also reverse the arrows and are not automatically forward stabilization maps.

5. Lie homology: separate the eventual theorem from the all-rank claim

Coefficients and the eventual statement

Definition 5.2 must require that the coefficient transition f_n intertwines the Lie transition i_n:

$$f_n(xm) = i_n(x) f_n(m).$$

If "maps" already means morphisms of these modules, this is an implicit convention to make explicit. If only graded equivariant linear maps are meant, there is a counterexample: take $L_n = \mathbb{Q}z$ in degree one, $M_n = \mathbb{Q}u \oplus \mathbb{Q}v$ in degrees zero and one, identity transitions, trivial group actions, and alternate $zu = v$ with $zu = 0$. All representation pieces are strongly stable, but $H_0(L_n; M_n)^1$ alternates between zero and $\mathbb{Q}$. Trivial and natural adjoint coefficients already have compatibility.

With compatible maps, finite-dimensional graded pieces and the finite uniform operation inputs, the eventual theorem has a replacement proof. Fix total degree d. Its Chevalley–Eilenberg complex has only finitely many chain terms. In the stable range write each chain term as a sum $S_{\lambda,n} \otimes A^i_{\lambda,n}$. Strong stability and matching branching multiplicity one make each transition $u_{\lambda,n} \otimes T^i_{\lambda,n}$ with T^i an isomorphism. The differential acts on the multiplicity spaces; the chain square makes the T^i chain isomorphisms. Homology therefore has injective, generating, type-preserving maps and constant finite multiplicities. Homology injectivity is proved here; it does not follow from arbitrary injections of complexes.

For reverse implications use the finite-degree Euler identity $\sum_i (-1)^i [C_i^d] = \sum_i (-1)^i [H_i^d]$ and induction on the Lie degree. With trivial or adjoint coefficients the unknown Lie piece occurs with the unit coefficient. For a general compatible coefficient take its least eventually nonzero grade and use the finite uniform cancellation proved above. This supports the intended finite eventual equivalence, relative to the specified operation and branching inputs. The symplectic multiplicity-only reverse statement has the same Euler argument with trivial coefficients; no symplectic map theorem follows.

Failure at the first nonzero rank

Definition 5.5 demands orbit generation at every rank. But $\bigwedge^2 \mathbb{Q} = 0$ cannot generate $\bigwedge^2 \mathbb{Q}^2 \neq 0$. This directly contradicts the stated conclusions of:

- Corollary 5.7 at free-Lie degree two;
- Corollary 5.8 at nilpotence class one and homological degree two;

- Corollary 5.9 at class one and homological degree one: the image in $E_2 \otimes E_2$ generates its symmetric square, missing its exterior square.

These are canonical inclusions, not artificial transition maps. They represent one definition-level defect, not three independent incorrect homology computations. For Theorem 5.6 the abelian Lie algebra $L_n = E_n$, trivial coefficients and $H_2 = \bigwedge^2 E_n$ give the same failure under the usual eventual type-preserving hypothesis. Reading constant-tail type preservation as an extra all-rank hypothesis changes this test at rank one, but does not rescue the three explicit corollaries or the proof's asserted all-rank equivalences.

There is a further small-rank error in identifying SIV with the unipotent test IV′. On $E_n \oplus \bigwedge^2 E_n$, change only the first map to $e_1 \mapsto (e_1, e_1 \wedge e_2)$. It is injective, orbit-generating, and its image is fixed by the last-column unipotents. The old standard line generates two new simple summands, so SIV fails.

A useful changed conclusion

Replace full orbit generation at a new type's first rank by injectivity, constant polynomial Schur multiplicities wherever defined, and preservation of each existing polynomial type. For such a sequence the old space identifies with the new space's weight-zero subspace for scaling the last coordinate. All weights are nonnegative; tensor and exterior operations preserve this identification, and compatible differentials preserve weights. It passes to homology.

Free Lie words, nilpotent truncations and their trivial/adjoint chain complexes have precisely this property: count occurrences of the new generator, and retract by killing it. This supplies a checked forward replacement for the applications. At a fixed degree only finitely many shapes occur, so eventual full strong stability follows once all those shapes exist.

This changed condition does not by itself prove the printed five-way equivalence. A reverse all-rank claim also needs polynomial control of the Lie maps and coefficient detection at the tested ranks. Astra supplied a further formal-series Euler argument using the whole coefficient module; it is retained as a separate conditional proposal, not needed for the forward replacement accepted here.

6. Prime two and the exterior-cube table

Write $A_g = H_1(\mathrm{Mod}_{g,1}(2); \mathbb{Z})$. Sato's 2008 Proposition 4.9, for the same one-boundary-component group and $g \geq 3$, states

$$A_g \cong (\mathbb{Z}/8)^{2g} \oplus (\mathbb{Z}/4)^{\binom{2g}{2}} \oplus (\mathbb{Z}/2)^{\binom{2g}{3}}.$$

Its exponent is eight, so it is not an $\mathbb{F}_2$ vector space. The theorem statement and group conventions were checked; Sato's whole proof was not audited.

A separate obstruction needs less: on a level-two symplectic matrix $M = I + 2X$, write X in blocks $(A, B; C, D)$. Then

$$f_i(M) = B_{ii} + 2\sum_j A_{ij}B_{ij} \pmod 4$$

is a surjective homomorphism. The symplectic relation makes B, C symmetric and $D = A^t$ modulo two. In a product, the defect in the linear term equals twice the polarization of the quadratic term; their sum vanishes modulo four. The transvection $I + 2E_{i,g+i}$ maps to one. Pullback along the standard surjection from the level mapping-class group gives a $\mathbb{Z}/4$ quotient. The surjectivity of the mapping-class action is an explicit imported premise. This already rules out an $\mathbb{F}_2$-vector-space structure.

Integral homology, $A_g/2A_g$, and the simple factors of a finite $\mathbb{Z}[Sp_{2g}(\mathbb{F}_2)]$-module are different objects. Changing between them changes the theorem. Putman's relevant extension is integral; tensoring it with $\mathbb{F}_2$ is not left exact.

The displayed exterior-cube table has its congruence cases interchanged. For $H = \mathbb{F}_p^{2g}$ let $\epsilon(x) = x \wedge \omega$ and let δ be symplectic contraction. Then

$$\delta\epsilon = (g-1)I.$$

For $g \geq 3$ with $g \equiv 1 \pmod p$, the injected H lies in $\ker\delta$, and the quotient by that kernel is another H. The printed table lists only one. This filtration alone disproves that row without assuming a formula for the remaining simple quotient's dimension.

The complete corrected class is

$$[\bigwedge^3 H] = [L(\omega_3)] + (1 + [g \equiv 1 \pmod p])[H].$$

The source route uses the Weyl-module formula in Baranov–Suprunenko Theorem 2.1, its characteristic-two extension in Corollary 2.9, the characteristic-independent character identity $[\bigwedge^3 H] = [W(\omega_3)] + [H]$, and restriction of p-restricted simple modules to the finite symplectic group. These premises and conventions are explicit; their complete proof histories are not newly certified. Fable also supplied an algebraic-group weight argument for the simple quotient. It must be done over an algebraic closure before finite-group restriction, since the split torus of the finite group at $p = 2$ cannot justify that weight argument.

With Putman's integral sequence and the quoted remaining factor calculations, the corrected integral $\mathbb{Z}[G]$ lists are $(2, 5, 2, 1)$ for odd g and $(4, 4, 2, 1)$ for even g, in the order $\mathbf{1}, H, L(\omega_2), L(\omega_3)$. These are conditional factor lists, not lists for $A_g/2A_g$, and they do not establish stabilization-map properties. For odd primes, a central $-I$ argument repairs the exponent inference in the equivariant extension: multiplication by p would give a map from an even central-character quotient to an odd central-character kernel, so is zero. Naturality of the required extensions under stabilization still needs checking before asserting the full corrected periodic representation-stability theorem.

Later work matters. Tudur Lewis's *Representation stability for the first homology of congruence subgroups*, arXiv:2507.20902v2 (1 June 2026), states corrected field-coefficient and uniform-periodicity results. Those statements were inspected, not their full proofs. They do not validate CF's original argument, and their existence prevents a claim that this issue has never been noticed or that the corresponding modern result remains wholly unsettled.

7. Successful checks, coverage and remaining work

Sol and Fable independently completed the braid packet. The argument uses at most $2i$ active labels in degree i, followed by induction from a fixed symmetric group and Hemmer's stable range $n \geq 2k$, giving $n \geq 4i$. Hemmer's original four-page article and its induction/Pieri proof were inspected. Earlier Orlik–Solomon inputs and every downstream twisted-cohomology assertion were not independently re-established.

Round	Model and effort	Completed mathematical assessments	Limit
Independent	Sol 6.2, high	4 / 4	Four assigned scopes; findings still needed adjudication
Independent	Fable 5.2, high	1 / 4	Foundations, Lie and modular attempts hit the 65,536 combined token cap
Focused	Astra 6.2, max	2 / 2	Non-blind foundations and Lie challenges
Focused	Fable 5.2, high	1 / 1	Non-blind modular challenge; partial supplied context and metadata recovery

The focused Fable answer's mathematical fields were preserved verbatim while source aliases and partial-access declarations were reconciled in a separate derivative. The original validation failure and its cost remain. Its packet mistakenly included a forward-reference page instead of Putman's Theorem 7.8; the coordinator subsequently checked the correct PDF page 29. That later check is not attributed to Fable. Missing theorem pages and full antecedent proofs remain different coverage questions.

Exact local checks passed for rational tensor and symmetric-square invariants at ranks 2–7, exterior maps at genera 2–8 over four prime fields, and 400 products of level-two symplectic matrices modulo eight. Their script and result hashes are preserved. Finite checks support conventions and examples; the general arguments above do the mathematical work.

The selected findings and finite uniform/forward polynomial repairs have now been consolidated. Remaining mathematical obligations are the full modular stabilization-map repair, exhaustive auditing of the classical imported inputs, and disposition of broader initial observations outside these focused targets. None is silently marked as verified. This publication preserves those limits; it does not claim a complete audit of the paper.

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