TECHNICAL READING REPORT

Flows, growth rates, and the veering polynomial

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4 November 2026

Version and scope. These notes concern **arXiv:2107.04066v3**, revised 5 August 2022; the downloaded PDF is dated 9 August 2022 and has 75 pages [1]. All internal page and theorem references below use that version. The published article is in *Ergodic Theory and Dynamical Systems* **43** (2023), 3026–3107. Its text has not been compared line by line with the arXiv version.

Assessment

The main geometric and symbolic constructions have a coherent purpose: they supply the transversality, control of coding multiplicity, and homotopies in the cut-open manifold needed to turn a graph calculation into a statement about flows. No counterexample to the transversality theorem or the closed-cycle coding theorem was obtained in this reading.

The principal unresolved issue is the strictness assertion in **Claim 9.11, p. 71**. The convergence furnished by Lemma 9.12 does not by itself produce a sequence of distinct growth rates. A simple reducible graph demonstrates the obstruction. A further example shows that irreducibility of the original graph, before restriction, does not remove it. Neither example is asserted to be a veering flow graph. The question is whether the geometry supplies an additional property, or whether the exact accumulation formula needs a different proof or qualification.

There is also a definite convention problem when cutting along a genuine fiber leaves no closed orbits: the displayed counting definition gives growth zero, the adapted polynomial is one, and the proposed logarithmic entropy is not a nonnegative real number. This is readily repaired, but the convention should be stated consistently.

The strictness question does not by itself invalidate Theorem E. The converse direction of Claim 9.11, together with the finite face structure and scaling by integral multiples, is sufficient for the headline compactness, reverse well-ordering, and derived-length bound. Section 2 explains this separation. The exact face-by-face equality and the criterion for maximal derived length deserve clarification.

This is a targeted mathematical review, not a certification of every local argument or of all cited external theorems. The text of the dynamic-plane, transversality, coding, growth and application arguments was examined; not every figure or imported foundational result was independently verified. The examples and repairs below are stated with their precise scope so that they can be evaluated independently.

1 The strictness step in Claim 9.11

Manuscript location: Claim 9.11, pp. 70–72, especially the paragraph after Lemma 9.12 on p. 71.

Write $C = \mathbb{R}_+\mathbf{F}$ for the fibered cone. For a face $\mathbf{S}$, let $D_{\mathbf{S}} = \Phi|\mathbf{S}$ be the recurrent graph formed by cycles annihilated by a class in the relative interior of its cone. The relevant set is

$$\Lambda(\mathbf{S}) = \{\mathrm{gr}(\alpha; D_{\mathbf{S}}) : \alpha \in \operatorname{int} C \cap H^1(M;\mathbb{Z})\}.$$

For $\mathbf{S} \subsetneq \mathbf{T}$, an integral $\eta \in \operatorname{relint}(\mathbb{R}_+\mathbf{T})$ and an integral $\alpha \in \operatorname{int} C$ with $\operatorname{gr}(\alpha; D_{\mathbf{T}}) > 1$, the argument gives

$$\operatorname{gr}(\alpha + i\eta; D_{\mathbf{S}}) \longrightarrow \operatorname{gr}(\alpha; D_{\mathbf{T}}). \tag{1}$$

The step needing justification is that proper containment of faces makes this a genuine accumulation sequence. Even a sequence that is nonconstant for a few initial terms can be eventually constant. What is needed is infinitely many distinct values approaching the limit.

1.1 Why reducibility matters

If a recurrent graph is the disjoint union of strongly connected components $J_1, \dots, J_r$, then

$$\operatorname{gr}(\alpha; D) = \max_j \operatorname{gr}(\alpha; J_j).$$

Changing the weights on some components may leave a different, dominant component unchanged. This is consistent with the componentwise maximum used in the entropy argument in §9.1.

For an explicit calculation, take $D = A \sqcup B$, with each of A and B a single vertex carrying two directed loops. Give both loops of A weight 1, and both loops of B weight $2 + i$. Put $D' = A$. A two-loop bouquet with common loop weight ℓ has growth $2^{1/\ell}$, since the number of cyclic words of length at most L/ℓ has that exponential rate. Therefore

$$\operatorname{gr}(\alpha_i; D) = \max\{2, 2^{1/(2+i)}\} = 2 = \operatorname{gr}(\alpha; D') \qquad (i \geq 0). \tag{2}$$

Weights are fixed on D', every cycle outside D' has weight tending to infinity, and the lower limit of the growth rates is greater than one. All the hypotheses of Lemma 9.12 hold. Its convergence conclusion is correct; the sequence supplies no distinct growth rates at all.

The corresponding polynomial makes the same point:

$$P_D^{\alpha_i}(u) = (1 - 2u)(1 - 2u^{2+i}).$$

The smallest positive root remains $1/2$. Sending the second factor toward one does not move that root.

1.2 What this establishes, and what it does not

This calculation disproves the proposed strictness inference for general reducible directed graphs. It does *not* prove that Claim 9.11 is false for the restricted graphs arising from this paper's three-manifold geometry. Appendix A strengthens the diagnostic by placing it inside a strongly connected ambient graph and a rational three-dimensional positive cone. A geometric realization of that model has not been supplied.

The requested clarification is accordingly quite specific:

> Which property of $\Phi|\mathbf{S}$ ensures that enlarging $\mathbf{S}$ properly to $\mathbf{T}$ changes the dominant growth rate, or otherwise supplies infinitely many distinct elements of $\Lambda(\mathbf{S})$ approaching every prescribed element of $\Lambda(\mathbf{T}) \setminus \{1\}$?

A sufficient condition for the displayed sequence would be a strict Perron comparison on all components that can realize the limiting maximum. For example, within a strongly connected component, suppressing a genuine part of its recurrent dynamics lowers the growth strictly. The issue is that a surviving whole

component can already realize the maximum while all affected components have smaller growth. Ambient irreducibility alone does not address this after cutting.

If a separate geometric lemma rules out this situation, it would complete the missing link. Otherwise the exact equality of derived sets should be distinguished from the weaker inclusion concerning actual accumulation points.

2 Conclusions that do not require that strictness step

Manuscript location: Converse half of Claim 9.11, p. 72; Theorem 9.10 and Theorem E.

There is a substantial amount that can be retained without the disputed direction. The following reconstruction uses the orbit–graph correspondence, positivity and face duality established earlier in the paper, and the corrected elementary argument for Lemma 9.12 in Section 4 below.

2.1 The actual-limit direction

Suppose pairwise distinct values

$$\lambda_i = \mathrm{gr}(\alpha_i; D_{\mathbf{S}}) \longrightarrow \lambda > 1$$

are given. Represent the integral fiber classes by nonnegative integral edge cocycles. After passing to a subsequence, each edge weight is either constant or tends to infinity. Let E be the subgraph formed by the constant-weight edges. Lemma 9.12 identifies the limit as the growth of E with its fixed weights.

As in the paper, one may first add $i\eta_{\mathbf{S}}$ without changing these growth rates, to force the weights of cycles outside $D_{\mathbf{S}}$ to diverge. The differences $\alpha_i - \alpha_1$ then vanish on cycles in E and are eventually positive on every other simple directed cycle. There are only finitely many simple directed cycles. Thus a single sufficiently large index works for all of them, and duality places the difference in the relative interior of the cone on a face $\mathbf{T}$ containing $\mathbf{S}$.

That containment is proper: otherwise the recurrent cycles would be unchanged and the growth values would be constant. The recurrent part of E has exactly the $\mathbf{T}$-null cycles. Consequently one obtains the inclusion

$$\left(\overline{\Lambda(\mathbf{S})}\right)' \setminus \{1\} \;\subseteq\; \bigcup_{\mathbf{T}\supsetneq\mathbf{S}} \Lambda(\mathbf{T}) \setminus \{1\}, \tag{3}$$

with $\mathbf{T}$ proper in $\mathbf{F}$. Since a subgraph cannot have larger growth than the full graph, the same argument gives $\lambda_i \geq \lambda$ on the extracted subsequence. If infinitely many terms of the original sequence lay below λ, applying the argument to that subsequence would be a contradiction. Thus the eventual inequality holds for the full sequence as well.

This direction requires an actual sequence of distinct values as input and never needs to manufacture one from proper inclusion of faces.

2.2 Compactness and reverse well-ordering

Let $K = \overline{\Lambda(\emptyset)}$. There is a uniform upper bound on its elements. For a nonnegative integral edge cocycle m positive on every directed cycle, replace every positive edge value by 1 and leave the zero values unchanged. The resulting cocycle m' is still positive on every cycle, satisfies $m' \leq m$, and has growth at least that of m. Only finitely many such $\{0,1\}$-valued cocycles exist. Taking their maximum gives a uniform bound. Closure then gives compactness in $[1,\infty)$.

There can be no strictly increasing sequence of growth values converging to a limit greater than one: equation (3)'s eventual inequality forces the values to approach that limit from above. The same applies to an increasing sequence in K, by approximating its terms with actual growth values in disjoint, ordered intervals. Any increasing sequence in K would have limit greater than one. Therefore K has no infinite strictly increasing sequence, which is equivalent to well-ordering under the reverse order $\geq$.

2.3 The derived-length bound and the final singleton

Put $d = \dim H^1(M;\mathbb{R})$, and let $K^{(j)}$ denote the jth derived set. Applying (3) repeatedly moves to a strictly larger face at every stage away from the value 1. There are finitely many faces, and the derived set of a finite union is the union of the derived sets. A chain beginning at the empty face has fewer than d nonempty proper faces of $\mathbf{F}$. It follows that

$$K^{(d)} \subseteq \{1\}.$$

One must also justify that the first such stage is $\{1\}$ rather than the empty set. All positive integral multiples of a fibered class are included in the paper's definition. Hence

$$\mathrm{gr}(r\alpha;\Phi) = \mathrm{gr}(\alpha;\Phi)^{1/r} \qquad (r \in \mathbb{N}).$$

The maps $x \mapsto x^{1/r}$ preserve K and, by continuity and injectivity, preserve each of its derived sets. If $K^{(j)}$ contains any $x > 1$, then it contains the distinct values $x^{1/r}$, which converge to 1. Thus $1 \in K^{(j+1)}$. At the first stage n with $K^{(n)} \subseteq \{1\}$, this proves

$$K^{(n)} = \{1\}, \qquad 1 \leq n \leq d.$$

For the unpunctured formulation, the face argument is applied in the original cohomology subspace after puncturing, as in the paper; the dimension bound refers to that subspace.

2.4 A further repair for the uncut flow

When $\mathbf{S} = \emptyset$, strictness can be supplied separately. Entropy on the full fibered cone is strictly convex, as recalled in the paper's Remark 9.2; see also the reciprocal-entropy formulation in [4, Corollary 5.4]. For a nonzero boundary class η and an interior class α, the function

$$t \longmapsto h(\alpha + t\eta), \qquad t \geq 0,$$

is nonincreasing: η is nonnegative on every closed orbit, so increasing t increases all counting weights. Strict convexity rules out a constant interval. Thus the function is strictly decreasing, and equation (1) produces distinct *global* growth values approaching each positive-entropy boundary growth value.

Together with the actual-limit direction and the power-root operation, this also supports the second additional assertion of Theorem 9.10: a nonzero boundary class annihilating infinitely many primitive orbits produces an accumulation value greater than one and hence infinitely many accumulation values. Conversely, an infinite derived set must contain a value greater than one, to which the actual-limit argument applies. This repair uses strict convexity for the full fibered cone and does not assume it for general cuts.

Impact. The headline conclusions of Theorem E, the description of every actual accumulation value greater than one as a cut-flow growth rate, and the preceding global criterion have available arguments. The exact all-face equality remains unresolved by these repairs. The necessary direction of the maximal-derived-length criterion follows from the face-chain argument; its sufficient direction still needs the missing strictness input or a replacement argument.

3 The empty-orbit case needs a convention

Manuscript location: Theorem A and equation (1.1), pp. 2–3; Theorem 7.2, p. 56; entropy definition and Theorem 9.1, p. 63.

Take a pseudo-Anosov suspension with singular orbits, remove those orbits, and let S be a genuine fiber of the resulting punctured mapping torus, carried by the layered veering triangulation. Every flow orbit meets S. Consequently the cut-open manifold has no closed flow orbits and its restricted recurrent graph is empty.

For every L the displayed counting function is zero, so its literal exponential growth rate is

$$\lim_{L\to\infty} \#\{\gamma \in \mathcal{O}|S : \xi(\gamma) \le L\}^{1/L} = \lim_{L\to\infty} 0^{1/L} = 0.$$

The positivity requirements are vacuous in this case. The Perron polynomial of the empty recurrent graph, and therefore the adapted veering polynomial, is 1. It has no positive root. Finally, $\log 0$ does not define a finite, nonnegative entropy.

This is an example within the intended class, rather than an arbitrary directed-graph diagnostic. It exposes three statements that cannot simultaneously be read literally: the raw counting definition, an unconditional smallest-positive-root formula, and an everywhere nonnegative real-valued entropy. The proof of Lemma 9.12 itself explicitly recognizes the acyclic graph convention $P = 1$, $\mathrm{gr} = 0$ on p. 73; the same convention also occurs in McMullen's graph treatment [2, p. 12].

Two consistent revisions are available:

1. Retain the raw growth definition, assign growth 0 to an empty count, and state the root and logarithmic-entropy assertions only in the nonempty case, with a separate empty-case clause.
2. Use normalized growth
$$\widehat{\mathrm{gr}}(\xi) = \lim_{L\to\infty} \max\{1, N_\xi(L)\}^{1/L},$$
so that the empty case has growth 1 and entropy 0. The polynomial statement must then separately assign this value when the polynomial is 1; it is still not the reciprocal of an existing positive root.

Whichever convention is chosen should also be used when comparing ordinary closed orbits with the augmented set containing prong curves. Removing finitely many primitive families and all their positive iterates preserves growth greater than one, but it can change a raw growth rate from one to zero if no family remains. The proof in §7.3 treats the growth-greater-than-one case correctly; the residual zero/one cases should be stated explicitly.

None of these revisions changes a positive exponential growth rate or the fibered stretch factors considered in Theorem E.

4 A local repair in the proof of Lemma 9.12

Manuscript location: Final proof of Lemma 9.12, p. 73.

Let $\lambda_i = \mathrm{gr}(\alpha_i; D)$ and $\lambda = \mathrm{gr}(\alpha; D')$. The hypothesis is $\liminf_i \lambda_i > 1$. At this stage one does not yet know that $\lambda > 1$; in particular, the proof has just allowed the possibility that D' is acyclic and $\lambda = 0$.

The needed inequality is simply $\mu > 1$ for any subsequential limit $\lambda_i \to \mu$, and this follows directly from the hypothesis. The rest of the polynomial argument can then be made precise without assuming the desired conclusion.

Only finitely many simple directed cycles occur. Their weights show that, for sufficiently large i, cycle weights dominate those of a fixed earlier class, so the λ_i are uniformly bounded above. The Perron polynomial expansion gives

$$0 = P^{\alpha}_{D'}(\lambda_i^{-1}) + N^{\alpha_i}(\lambda_i^{-1}),$$

where every term of $N^{\alpha_i}(t^{-1})$ has the form at^{-x_i} with $x_i \to \infty$. Because $\lambda_i \to \mu > 1$, the second summand tends to zero. Hence

$$P^{\alpha}_{D'}(\mu^{-1}) = 0.$$

If D' were acyclic, its polynomial would be 1, a contradiction. Otherwise λ is the largest positive reciprocal root, so $\mu \leq \lambda$. Subgraph monotonicity gives $\mu \geq \lambda$, proving $\mu = \lambda > 1$.

This is a local correction to the order of the reasoning. The convergence lemma survives it. It does not add the strictness needed in Section 1.

5 Checks on the main proof architecture

The following points explain why the earlier, longer portions of the paper should be evaluated separately from the final accumulation argument.

5.1 Transversality is a substantive construction

A combinatorial triangulation associated to a flow does not automatically yield an embedded realization whose faces are transverse to that flow. The argument in §5 addresses this through an equivariant realization over flow space, control of the maps on the line fibers, straightening, and smoothing. The convolution step has a useful quantitative check. With a suitable symmetric kernel ρ, its derivative is controlled by an integral of

$$k(t+u) - k(t-u),$$

over a fixed range of positive u. Coarse monotonicity makes this uniformly positive. Translation invariance of the construction preserves equivariance. Thus this part supplies an actual monotone fiber map. This check is conditional on the uniform estimates in Lemma 5.12. The compactness-to-uniform-leaf-diameter passage in that lemma, particularly at compactification boundaries, is compressed; its full topological justification was not independently reconstructed here. No counterexample to that estimate was found. The anchor choices, properness and global gluing are separate requirements in the construction.

5.2 Uniform coding is the relevant strength

For growth comparison, an arbitrary finite-to-one correspondence would be insufficient: the size of the fibers could itself grow exponentially. What is needed is a uniform multiplicity bound together with control of the exceptional orbit families. The chain from adjacent AB strips to consecutive nonhinge tetrahedra in a single fan gives the relevant uniform width bound. For nonsingular point-lines, the finite chain gates can be placed before all mergers in any given finite family; deterministic forward continuation then bounds that family's size. These are concrete mechanisms supporting the multiplicity estimates.

For any fixed positive class ξ, the positive iterates of a fixed primitive orbit γ contribute at most $\lfloor L/\xi(\gamma) \rfloor$ terms of weight at most L. A finite union of such families contributes $O(L)$. If $N(L)$ has exponential rate $g > 1$, then replacing it by $N(L) + O(L)$ or changing it by a uniformly bounded

multiplicity leaves g unchanged. This verifies the counting reduction once the stated coding properties and homotopies are in place. It also explains precisely why the empty case must be handled separately.

One scope limitation should be explicit. Proposition 6.12 gives its numerical bounds at nonsingular points, while the introductory Theorem D includes flow lines at singular orbits. The ladder discussion controls the closed-cycle contribution there; a bound for all potentially nonperiodic bi-infinite lines mapping to a singular orbit was not independently established in this review. This is an unverified extension of the introductory formulation, not a counterexample to the closed-cycle result used for growth.

5.3 The cut-open homotopy is essential

The growth parameter belongs to $H^1(M|S)$ and need not extend from M. Therefore a homotopy between a graph cycle and a flow orbit merely in M would not suffice. The positive transverse homotopy used in §7 stays in the appropriate cut component: intersection numbers with the carried surface are nonnegative, and their total is zero. This is the correct mechanism for evaluating the same cut-open cohomology class on both objects. The extra boundary curves and stronger positivity in §8 address the related difficulty for closed transverse surfaces in the original closed manifold.

5.4 The spectral calculation respects reducibility

Positivity on every directed cycle permits a coboundary adjustment to a positive edge metric, as in McMullen's Theorem 5.1 [2]. Thus real cohomology classes can be treated using the corresponding exponential-polynomial specialization; integrality is not needed at this step. On a reducible recurrent graph the entropy is the maximum of its component entropies. A finite maximum preserves continuity and convexity but does not generally preserve analyticity or strict convexity.

The stronger entropy argument correctly seeks a single component carrying the positive entropy and an injective pullback on cohomology. The latter matters: pulling a strictly convex function back along a map with a kernel can destroy strict convexity. These are the appropriate issues to check, and they are explicitly addressed in the detailed argument.

6 Small statement and notation clarifications

Preserving traversal multiplicity in §6.3, p. 53. When $c = c_0^k$, the lifted image line is the same as for c_0, but the deck element for one traversal is the kth power. A generator of the *setwise* stabilizer of that image line does not retain this information. The definition of $\mathfrak{F}(c)$ should use the deck element determined by the chosen traversal, or explicitly impose $\mathfrak{F}(c_0^k) = \mathfrak{F}(c_0)^k$. The intended homotopy-preserving correspondence is then clear. Related stabilizer language in Lemmas 3.14 and 6.16 should use the same convention. Likewise, a prong curve in §8 must map to the appropriate positive iterate of the singular orbit if one return permutes its prongs.

Width in the proof of Theorem 6.1, p. 54. Distinct periodic point-lines that represent preimages of a fixed orbit cannot be asymptotic, so the argument gives $n \le \mathrm{width}(D_p) \le \delta$. This inequality is sufficient. The printed equality with width would additionally need all asymptotic classes to be fixed by the chosen deck period; the annulus/Möbius discussion allows permutation of classes. No particular geometric counterexample to that equality is asserted here.

Theorem B versus Theorem 9.3. The introductory stronger entropy conclusion on p. 3 omits the condition that entropy is not identically zero, which appears in the detailed Theorem 9.3 on p. 64. It would be useful either to repeat that condition or to explain why it is redundant under the stated geometric hypotheses. No geometric counterexample to the introductory formulation is asserted here; a constant zero function by itself would not have the claimed strict properties on a nontrivial cone.

Augmented orbit count in §7.3. In the first counting comparison in the proof of Theorem 7.1, p. 58, the codomain should include the prong curves until the next paragraph removes their iterates. Writing $\mathcal{O}^+$ at that intermediate point would match the map actually being counted. The subsequent $O(L)$ argument supplies the intended comparison for growth greater than one.

Integral multiples and the apex in Theorem 9.10. The set of classes used there includes nonprimitive integral classes, corresponding to possibly disconnected fibers. This is consistent with the power-root scaling used above. Restricting the theorem to connected primitive fibers would change the set under discussion. In rank one, the present convention gives the useful check $\{\lambda^{1/r} : r \geq 1\}$, whose closure has derived set $\{1\}$. The second additional assertion of Theorem 9.10 should specify a *nonzero* boundary class, unless the boundary is intended in a projectivized sense. Under the ordinary topological-boundary convention, 0 is on the boundary and annihilates every orbit, making the condition automatic even in this rank-one example. No explicit exclusion of the apex was found in the surrounding discussion.

Claim 5.9, p. 35. The limit written as $a \to \infty$ in the middle of the proof should be $a \to 1^+$, consistently with the surrounding argument and conclusion.

Questions that would settle the remaining issues

1. In Claim 9.11, which additional property of restricted veering flow graphs supplies strict growth change, or what alternative sequence proves that every value on the right-hand side is a genuine accumulation point?
2. If the general face-by-face equality needs qualification, which parts of the maximal-derived-length criterion remain valid? The one-sided argument above already supplies the headline bound.
3. Which empty-orbit normalization is intended for the root formula and the entropy function, including the genuine-fiber case?
4. Should Theorem B repeat the nonzero-entropy hypothesis of Theorem 9.3, and should the local proof and notation corrections above be incorporated?

A A strongly connected ambient graph does not suffice

This example is an abstract graph-and-cone model, **not a claimed veering triangulation or three-manifold counterexample**. Its purpose is to test whether ambient irreducibility and a positive rational cone alone repair the strictness step.

Take two vertices a, b, two loops at each vertex, and one directed connector in each direction. Assign weight x to each a-loop, y to each b-loop, z to the connector $a \to b$, and zero to $b \to a$. The entire graph is strongly connected. Its integral homological labels span $\mathbb{Z}^3$, and positivity on all directed cycles is equivalent to

$$x > 0, \qquad y > 0, \qquad z > 0.$$

Zero weight on one connector causes no problem: a vertex coboundary can distribute the total z equally between the two connectors. The weighted adjacency matrix and its Perron polynomial are

$$A(u) = \begin{pmatrix} 2u^x & u^z \\ 1 & 2u^y \end{pmatrix}, \qquad P(u) = (1 - 2u^x)(1 - 2u^y) - u^z.$$

In the closed positive octant, let the cone on $\mathbf{S}$ be the z-axis and the cone on $\mathbf{T}$ be the face $x = 0$. The latter properly contains the former. Null-cycle restriction at $\mathbf{S}$ removes the connectors and leaves both two-loop bouquets. Restriction at $\mathbf{T}$ leaves only the a-bouquet.

For $\alpha = (1, 2, 1)$ and $\eta = (0, 1, 1) \in \operatorname{relint}(\mathbb{R}_+\mathbf{T})$,

$$\operatorname{gr}(\alpha + i\eta; D_{\mathbf{S}}) = \max\{2, 2^{1/(2+i)}\} = 2 = \operatorname{gr}(\alpha; D_{\mathbf{T}})$$

for every $i \geq 0$. More strongly, if the integral interior classes range over all positive integer triples, then

$$\Lambda(\mathbf{S}) = \{2^{1/n} : n \in \mathbb{N}\} = \Lambda(\mathbf{T}).$$

Thus

$$(\overline{\Lambda(\mathbf{S})})' = \{1\}, \qquad \Lambda(\mathbf{T}) \setminus \{1\} \neq \varnothing.$$

In this model even the proposed inclusion of sets of accumulation values fails, not just the particular sequence chosen to prove it. Additional properties special to the geometric setting are therefore necessary if they are to supply the missing argument.

References

[1] M. P. Landry, Y. N. Minsky, and S. J. Taylor, *Flows, growth rates, and the veering polynomial*, arXiv:2107.04066v3 (5 August 2022), arxiv.org/abs/2107.04066v3. Published in *Ergodic Theory Dynam. Systems* **43** (2023), 3026–3107, doi:10.1017/etds.2022.63.

[2] C. T. McMullen, *Entropy and the clique polynomial*, *J. Topol.* **8** (2015), 184–212, doi:10.1112/jtopol/jtu022. Author manuscript: perron.pdf.

[3] M. P. Landry, Y. N. Minsky, and S. J. Taylor, *A polynomial invariant for veering triangulations*, *J. Eur. Math. Soc.* **26** (2024), 731–788, arXiv:2008.04836. The Perron-polynomial identification and cone statements used by the reviewed paper were checked against their stated formulations; their proofs were not independently re-established in full here.

[4] C. T. McMullen, *Polynomial invariants for fibered 3-manifolds and Teichmüller geodesics for foliations*, *Ann. Sci. École Norm. Sup.* (4) **33** (2000), 519–560, doi:10.1016/S0012-9593(00)00121-X.